Location Privacy via Private Proximity Testing

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Abstract

We study privacy-preserving tests for proximity: Alice can test if she is close to Bob without either party revealing any other information about each other's location. We describe several secure protocols that support private proximity testing at various levels of granularity. We introduce the concept of location tags generated from the physical environment in order to strengthen the security of proximity testing. We implemented our system on the Android platform and report on its effectiveness. Our system uses a social network (Facebook) to manage user public keys. We argue that for proximity testing, social networks are better suited for managing user keys than traditional PKI.

1 Introduction

Location-aware devices, mainly smartphones, are ubiquitous and location-based services are proliferating rapidly. Privacy concerns are a major factor holding up the growth of this market [32, 33] and new regulations are being debated [29]. Users take location privacy seriously because of the possibility of physical harm [35]. For an analysis of the prevalence of location data on the Internet and the dangers posed by it, see [12].

Current location-based services, whether provided by carriers or by third-party applications, require the user to constantly transmit their location to a server (or at least whenever services are required). Service providers have had a poor track record of respecting users' location privacy [38] and user trust in the privacy of location-based services is therefore understandably low. It would be of great benefit to design practical location-based services in a way that minimizes the information transmitted to the service provider.

We consider the problem of *proximity testing with privacy* in the context of location-based social networking, which is one important and burgeoning type of location-based service. Private proximity testing enables a pair of friends to be notified when they are within a threshold distance of each other, but otherwise reveal no information about their respective locations to anyone.

Location-based social networking is broader than just proximity detection. For example, users might want activity streams filtered by location, location-based gaming, etc. We seek to construct the fundamental cryptographic primitives on top of which such functionality can be built in a privacy-preserving way. While proximity detection by itself covers a broad range of use-cases (we give several examples below), our solutions are also useful for the techniques that can be used to construct other types of privacy-preserving functionality.

We make the following contributions:

1. We put forth a set of desiderata for privacy preserving proximity testing. This is tricky because there are several desirable security and efficiency properties that conflict with each

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other. We aim for rigorous cryptographic security definitions. In Section 6 we explain why this is necessary and why more ad-hoc definitions are inadequate.

2. We reduce private proximity testing to the underlying cryptographic problem of private equality testing (PET). We consider this problem in two different settings: with or without the involvement of the server. The server-mediated setting is more efficient, but is not secure against the collusion of the server with the user's friends. While PET has been considered before, we design protocols in both settings that use less bandwidth.

3. We show how to use *location tags* to enable private proximity testing. A location tag is an ephemeral, unpredictable nonce associated with a location and can be derived from various electromagnetic signals available in the environment, such as WiFi and Bluetooth. It can be thought of as a shared pool of entropy between all users at a given location at a given time.

4. We describe a prototype that we have built for the Android platform. One implementation issue is key agreement between pairs of users. We take a novel approach to this problem: we designed and implemented a key agreement system using a social network as a key-distribution mechanism, rather than traditional PKI. Our system binds the user's public key to the user's social network account (the current implementation uses Facebook). We argue that using a social network in place of the traditional PKI approach is highly effective for managing user public keys.

Motivation. Let us consider several applications of proximity testing, keeping in mind that different applications require different proximity granularity.

- Alice and Bob are friends, and are serendipitously notified that they are shopping in the same mall. They meet and have a pleasant time together. Alternatively, Alice and Bob first meet online, but later decide to meet in person at a coffee shop. Alice arrives first and is notified when Bob arrives.

- Alice would like to get dinner with her friend Bob who travels a lot. Using privacy-preserving proximity testing, Alice can check if Bob is town before calling him. Note that for this application the proximity granularity is a wide geographic area.

- Bob, a student lands at his college airport and wants to check if anyone from his college is currently at the airport and can give him a ride to campus.

- Alice is a manager who wants to automatically record who is present at her daily meetings. However, her employees do not want their location tracked. Privacy-preserving proximity testing over this well organized group allows satisfying both requirements.

Using our system, existing location-based social networks such as Loopt and Google Latitude could offer a basic or 'lite' version with limited functionality but respecting privacy. This may spur adoption by privacy-conscious users, and make the product more useful overall due to the positive network externality (i.e., even the users who don't care about privacy benefit from having more friends in the system).

As discussed above, proximity detection is particularly useful for group meetings, either ad-hoc groups or well organized groups such as project teams. More esoteric applications may include unit detection in ground combat. Privacy ensures that in case of capture, the proximity test does not reveal the location of all combat units.

2 Model

All communication in our system takes place over the Internet, i.e., we do not make use of direct physical channels between nearby devices (e.g., Bluetooth). Not only do all location-based services today operate in the client-server model, the peer-to-peer model is fundamentally not capable of supporting the functionality that we desire, as we explain in Section 6.

Our system requires the existence of a social network, i.e., a graph that captures trust relationships between users. Our protocols allow detection of proximity between any two users connected by an edge; we assume the existence of shared secret keys between connected users (more details on this are in Section 5.1).

Proximity detection between strangers is a useful functionality, but it is not possible to do in a privacy-preserving way in the client-server model. The reason is that either the server will need to learn some information about users' locations, or it will need to treat every pair of users identically, resulting in overall bandwidth requirements quadratic in the number of users, unless limited to pairs of friends. As we discuss in Section 6, revealing even a minimal amount of information about users' locations (e.g., the single-bit outcome of proximity testing between pairs of users) to the server results in an unacceptable privacy leak when aggregated over time and users.

The ideal functionality of our model is in Section 2.5. When location tags are available, the model is somewhat different, and the ideal functionality is in Section 4. But first let us discuss some of the desiderata that will motivate our model.

2.1 Desiderata: Functionality

Asymmetry. Proximity testing is asymmetric: one party will learn if the other party is nearby whereas the other party learns nothing. This is necessary because asymmetric edges are common in social networks — Alice may be willing to let Bob test proximity to her, but not vice versa. Of course, if an edge is symmetric, then it can be treated as a pair of directed edges, and we can execute the protocol twice in either direction.

One important side-benefit of asymmetry is the some of our protocols can be executed in an *asynchronous* manner. This has the potential to greatly decrease the communication cost. We explain this in more detail in Section 2.4.

Proximity threshold. The distance threshold for proximity detection should not be globally fixed but instead configurable by each user. This is because a larger threshold is neither strictly worse nor strictly better than a smaller one, either from the security or the functionality perspective. With a larger threshold, the user is easier to locate but in case of a match their location is revealed less accurately.

Ideal functionality. The obvious way to define the "ideal functionality" is as follows: whenever Alice and Bob are within a distance δ (defined by Alice) of each other, Bob outputs 1, otherwise he outputs 0. However, there is a problem with this definition: even when δ is large, Bob can determine Alice's exact location by moving around and applying "triangulation" (assuming Alice is stationary).[1]

Since the natural ideal functionality is flawed, we must necessarily "quantize" the space of locations. We describe our quantization technique in Section 2.5.

[1] Loopt has reported that users often carry out this attack on their system.

2.2 Desiderata: security

The adversary might be law enforcement coercing the service provider into revealing the user's location, or it might be someone colluding with the service provider. It could also be one of the user's friends – either because the friend's account has been compromised, or the attacker set up a fake profile, or simply because friendship on social networks does not imply a high level of trust. Stalking is an example of the threat from this type of adversary.

Broadly, these break down into untrusted server and untrusted friend(s). Our overall goal is to reveal as little as possible to each party while enabling proximity testing. We now discuss each threat in more detail.

Honest-but-curious friend. The space of possible locations has low entropy, and is therefore vulnerable to a brute-force or dictionary attack. Suppose that the threshold distance for proximity detection is 10 meters; this results in a search space of roughly 10^{10}, which is less than 34 bits. Therefore, the only meaningful way to define privacy is in a manner analogous to *semantic security* or *indistinguishability* in cryptography; roughly, the protocol should reveal no information other than the output of the computation.

Malicious friend. If the attacker has some background information about the user, the search space becomes smaller. In the extreme case, the attacker might only be interested in answering a binary question, e.g., whether the user is at home or at work, in which case the search space is only 1 bit.

This shows another weakness of the ideal functionality: even a protocol that reveals nothing but the output of the computation is vulnerable to an online guessing attack by a malicious friend (who is willing to lie about his own location). Resisting such an attack is not critical: as we explain in the next subsection, protocols in our system are naturally rate-limited. Nevertheless, it is a useful desideratum. In Section 4 we explain how to resist online attacks using location tags.

Server. In some of our protocols, the server is treated as just a router of messages between users. These protocols are secure against a malicious server due to the existence of shared secret keys between friends, which are used to establish an encrypted and authenticated channel between the parties. The server can of course refuse to deliver messages, but can do no other harm.

In our most efficient protocols the the server acts as a participant. Here, we require that the server be able to learn no information about users' locations, even if it may be able to cause the users to compute an incorrect answer.

Collusion. Malicious friends may collude with each other; the security requirements should ideally still hold. As for the server, in protocols where it acts a router security should hold even if it collude with any of the user's friends. On the other hand, when the server is a participant in the protocol, we do not expect to have privacy against the collusion of the server with a friend (indeed, it is this very relaxation that allows efficiency gains in this model).

2.3 Desiderata: Efficiency

Mobile devices are resource-constrained, so we would like to construct solutions that minimize computation and bandwidth. Since we have an eye toward implementation, we go beyond asymptotic analysis and also focus on optimizing the constant factors involved.

Computation. While any two-party functionality can be securely computed by expressing it as a special case of Yao garbled-circuit evaluation [41], this is too inefficient. In general, we seek to

minimize the number of modular exponentiations and other expensive operations. Protocols that avoid the use of large groups altogether are particularly attractive.

Bandwidth is perhaps the most constrained resource, and we would like to minimize the bandwidth required per edge. It is an intriguing question whether we can use amortization to design a protocol whose bandwidth requirement is asymptotically smaller than the number of edges. It does not appear to be possible to do so unless we compromise on the security requirements.

Number of connections. Proximity testing needs to be carried out between each pair of friends; however, executing an instance of the protocol independently for each friend would involve opening multiple connections to the server (one for each edge in the system) and quickly becomes infeasible.

Instead, in our system, each user (say Alice) sends a single message to the server that encapsulates her messages from all the instances of the protocol – one for each friend. The server de-multiplexes these messages and then re-multiplexes them by recipient. In this manner, the number of connections grows asymptotically as the number of nodes in the system.

There are added subtleties in the case of synchronous protocols; details are in Section 2.4.

2.4 Synchronous vs. asynchronous execution

A proximity testing protocol with a *synchronous* communication pattern requires both parties to be online at the same time. The role of the server is merely message passing.

In the synchronous pattern, the execution of the protocol needs to happen at globally fixed time intervals (say once every 5 minutes). We call these intervals *epochs*. Each protocol round is executed synchronously by all users participating in the system, so that the multiplexing of messages described in Section 2.3 can happen.

On the other hand, the communication pattern in some protocols is *asynchronous*, which means that the two parties do not have to be online at the same time. In other words, it can be seen as a two party protocol between the "sender" and the server, followed by a two-party protocol between the "receiver" and the server.

In this setting, the server needs to be involved in the computation rather than being simply used for message passing. The reason is that if the server is only used for message passing, then it is equivalent to a two-party non-interactive protocol. This makes the message essentially a hash of the location, which allows a dictionary attack on the location. In practice the communication pattern always looks as follows: $B \to S; A \to S; S \to A$.

The asynchronous setting allows a privacy-efficiency tradeoff due to the fact that the sender and receiver can each execute their half of the protocol with the server at arbitrary times. Specifically, a user might configure her device to participate in the protocol in the role of sender only when her location changes. Similarly, she might configure her device to participate in the role of receiver only when she explicitly checks the proximity testing application.

The detriment to privacy comes from the fact that the server learns when the user is moving. This is of course a much less severe leak than leaking the location. Nevertheless, the server might be able to tell, for example, when a user went home for the night. It is important to note that each user can control their participation schedule and change it at any time.

2.5 Reducing proximity testing to equality testing.

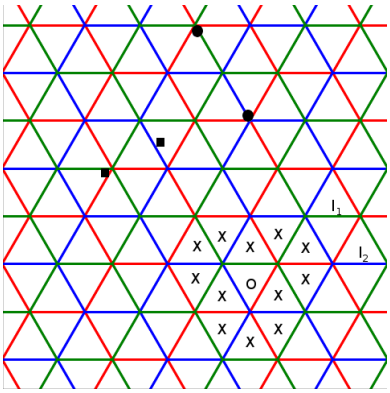

Figure 1: The triangles represent minimally separated users who are not part of the same cell in either grid. The circles represent maximally separated users are in the same cell in one of the grids (the red one).

Let G be a grid or a tessellation of the plane,[2] and let L_u be the location of user u expressed as the center of the grid cell that the user is located in. For any two users u and v, the equality testing protocol $\Pi_{u,v}$ must satisfy the following:

- if $L_u = L_v$ then u learns L_v
- if $L_u \neq L_v$ then u learns nothing about L_v except that $L_u \neq L_v$

By running any such protocol Π on three hexagonal grids $\mathsf{G}_1, \mathsf{G}_2, \mathsf{G}_3$ which are mutually offset in the manner shown in Figure 1, we obtain a proximity testing protocol Π' such that

- if $||X_u - X_v|| \leq \delta$ then u learns at least one of $L_{v,1}, L_{v,2}$ and $L_{v,3}$.
- if $||X_u - X_v|| \geq \gamma\delta$ then u learns nothing about X_v except that $||X_u - X_v|| > \delta$

where $L_{u,i}$ is the quantized location of user u in grid i, δ is half the height of each hexagonal cell, γ is the diagonal of the hexagons and X_u is the exact location of a user u. Π' is the ideal functionality for proximity testing.

Intuitively, what this means is that (i) proximity testing only reveals quantized locations, (ii) it is guaranteed to do so when the parties are sufficiently close, and (iii) guaranteed not to do so when they are sufficiently far apart.

The values δ and $\gamma\delta$ are respectively the closest two users can be and not be detected as nearby, and the farthest they can be and still be detected as nearby. Note that the value of γ is $4/\sqrt{3}$.

The value of $\gamma\delta$ is visually clear. To derive the value of δ, we argue as follows: let A and B a pair of such closest points not detected as nearby. Without loss of generality, A lies in the triangle marked O. B cannot lie in the region marked X, and therefore A and B are separated by at least one of six pairs of parallel lines (one such pair (ℓ_1, ℓ_2) is shown in the diagram).[3]

Our choice of grid is the simplest one that works, because at least three overlapping grids are required. (With only two grids, consider any point at which they intersect. Then there can be two people arbitrarily close near this point who will not be detected as nearby.)

3 Private Equality Testing

The private equality testing problem was studied in a number of papers [10, 26, 6, 20]. Here we describe two concrete protocols that are especially well suited for our purposes. They solve the following problem:

Input: Alice has value a representing her location. Bob has value b representing his location.
Output: Alice learns if $a = b$ and nothing else. Bob learns nothing.

[2] more accurately, the surface of the Earth. In Section 5 we describe how we apply a grid to a curved surface.
[3] The diagram is likely to be unclear if printed in monochrome.

We call this problem *asymmetric* equality testing because Alice learns the answer but Bob does not. This sort of asymmetry is common in social networks. Our first protocol is computationally more expensive than the second, but provides stronger security guarantees.

3.1 Protocol 1: Synchronous private equality testing

In this protocol the server is used only to forward messages between the two parties, and does not perform any computation. The protocol is synchronous meaning that both parties need to be online to carry out the protocol.

The protocol is based on a mechanism of Lipmaa [20] and has the following characteristics: (1) each party performs either 2 or 3 exponentiations, (2) the protocol is two rounds, namely Alice sends a message to Bob (through the server) and Bob responds to Alice, (3) communication is about 40 bytes per edge per time interval using elliptic curves of size 160 bits (additional end-to-end encryption introduces a negligible overhead). The protocol is secure against arbitrary collusion assuming the standard Decision Diffie-Hellman problem. The protocol proceeds as follows.

Global setup: Let G be a cyclic group of prime order p and g a generator of G. We will assume that the Decision Diffie-Hellman problem is hard in G. All clients in the system are pre-configured with the same G and g. In what follows we will use $\mathbb{Z}_p$ to denote the set $\{0, \ldots, p-1\}$.

Client setup: When the system is first installed on the client it chooses a random x in $\mathbb{Z}_p$ and computes $h \leftarrow g^x$. Thus, Alice has x and h and h will be used as her ElGamal pubic key. We assume Bob already has Alice's public key h (more on this in Section 5.1).

Round 1: Alice computes an ElGamal encryption of her location a encoded as h^a and sends the resulting ciphertext to Bob (through the server). More precisely, Alice chooses a random r in $\mathbb{Z}_p$, computes

$$C_{\mathrm{a}} \leftarrow (g^r,\ h^{a+r})$$

and sends C_{a} to Bob.

Round 2: Bob chooses a random non-zero s in $\mathbb{Z}_p$ and uses his own location b and the ciphertext $C_{\mathrm{a}} = (g_1, g_2)$ from Alice to construct a fresh ElGamal encryption of the message $s \cdot (a-b)$. More precisely, Bob chooses a random t in $\mathbb{Z}_p$ and computes

$$C_{\mathrm{b}} \leftarrow \left(g_1^s\, g^t,\ \ g_2^s\, h^{(t-sb)}\right)$$

Observe that setting $w := sr + t$ we have

$$C_{\mathrm{b}} = \left(g^{sr+t},\ h^{s(a-b)+sr+t}\right) = \left(g^w,\ h^{s(a-b)+w}\right)$$

so that C_{b} is a fresh encryption of $s(a-b)$ under the public key h. Bob sends C_{b} to Alice through the server.

Obtain answer: Alice now has $C_{\mathrm{b}} = (u_1, u_2) = (g^w,\ \ h^{s(a-b)+w})$ and her secret key x. She decrypts C_{b}, namely computes $m \leftarrow u_2/u_1^x = h^{s(a-b)}$. If $m = 1$ she concludes that $a = b$ and if not she concludes that $a \neq b$.

Security. The protocol above is an optimization of the generic mechanism of Lipmaa who provides a proof of security [20, Theorem 4]. We briefly sketch the argument: Alice's privacy is assured under

the DDH assumption since the only thing she sends to Bob is an ElGamal encryption of her location. Bob's privacy is assured unconditionally since all Alice learns is $s(a-b)$ which is either 0 if $a=b$ or random non-zero in $\mathbb{Z}_p$ if $a \neq b$. When $a \neq b$ this reveals no other information about b.

Performance. Since computing a product of exponents such as $g_1^s g^t$ is not much more expensive than computing a single exponent (see [24, p. 619]) we count these as a single exponentiation. Overall, Alice performs three exponentiations and Bob does two. Two of Alice's exponentiations use a fixed base which can be sped up considerably using pre-computations. The total traffic amounts to four ElGamal ciphertexts which is about 320 bytes using 160-bit elliptic curves.

3.2 Protocol 2: Fast asynchronous private equality test with an oblivious server

Our second private equality test, shown in Figure 2, is novel and requires far less communication and computation, but is only secure assuming the server does not collude with either party. The server learns nothing at the end of the protocol. The reason for the performance improvements is that this protocol uses three parties (Alice, Bob, and server) and is therefore able to rely on information theoretic methods such as secret sharing.

We describe the protocol as it would be used in our system, namely at time t Alice is in position a_t and Bob is in position b_t. At various times t Alice wants to learn if $a_t = b_t$ without learning anything else about b_t. As mentioned above, the server and Bob learn nothing.

Global setup: Let p be a prime so that all possible locations are in the range $[1, p]$. If location data is 32 bits then one can set $p := 2^{32} + 15$. All clients in the system are pre-configured with the same p. As before we let $\mathbb{Z}_p$ denote the set $\{0, \ldots, p-1\}$.

Client setup: When Alice and Bob declare themselves as friends they setup a shared secret key which we denote as k_{ab}. We explain key setup in more detail in Section 5.1. Both clients also maintain a counter ctr which is initially set to 0. We also assume that when signing up to the service, Alice and Bob each generate a secret key with the server denoted k_{a} and k_{b} respectively.

The keys $k_{\text{ab}}, k_{\text{a}}, k_{\text{b}}$ will be used as keys to a Pseudo Random Function (PRF) denoted by $F(k, x)$, where k is the PRF key and x is the point at which the function is evaluated. (Our implementation uses AES as the PRF.) All communication between Alice and the server and Bob and the server is encrypted with an authenticated encryption scheme.

Step 1 (Bob's message): Bob increments ctr by one and computes $(k_1, k_2) \leftarrow F(k_{\text{ab}}, \text{ctr})$ and $r \leftarrow F(k_{\text{b}}, \text{ctr})$. Bob parses the result so that k_1, k_2, r are elements in $\mathbb{Z}_p$ and sends $m_{\text{b}} \leftarrow r(b_t + k_1) + k_2$ and ctr to the server.

Step 2 (Alice queries server): When Alice wishes to query the server she increments her ctr by one[4] and computes $(k_1, k_2) \leftarrow F(k_{\text{ab}}, \text{ctr})$. Alice parses the result so that k_1, k_2 are elements in $\mathbb{Z}_p$ and sends $m_{\text{a}} \leftarrow a_t + k_1$ and ctr to the server.

Step 3 (Server responds to Alice): The server finds a message from Bob that has the same counter value ctr as the message from Alice. It computes $r \leftarrow F(k_{\text{b}}, \text{ctr})$ and parses the result as an element in $\mathbb{Z}_p$. It sends to Alice the message

$$m \leftarrow r\, m_{\text{a}} - m_{\text{b}} \qquad = r(a_t + k_1) - r(b_t + k_1) - k_2 = r(a_t - b_t) - k_2$$

Alice computes $m + k_2 = r(a_t - b_t)$. If the result is 0 then she knows $a_t = b_t$ and otherwise not.

[4] In our implementation, instead of incrementing ctr, there is an extra step preceding this where Alice first queries the server to obtain the latest value of ctr (for each friend).

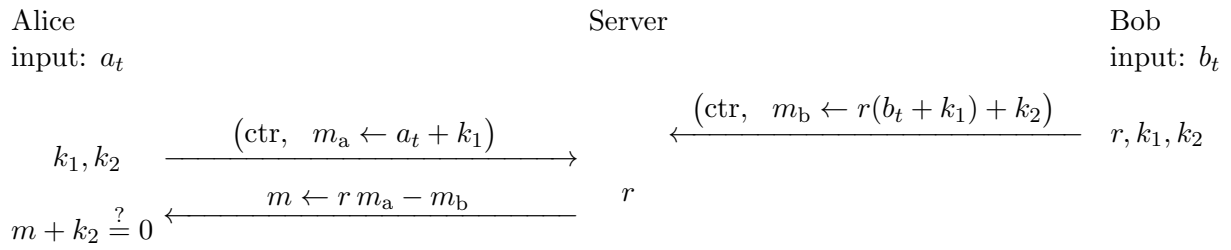

Figure 2: Asynchronous equality test with an oblivious server

Security. We show that the protocol is secure (i.e. no party learns more than it should) as long as no two parties collude. First, since F is a secure Pseudo Random Function, the outputs k_1, k_2, and r at every iteration are indistinguishable from truly random and independent values in $\mathbb{Z}_p$.

Now, observe that Bob learns nothing from the protocol since he recieves no messages. The server sees m_a and m_b and both are independent of the users' inputs a_t and b_t. To see why, recall that m_a is blinded by k_1 and m_b is blinded by k_2. Since k_1 and k_2 are independent random values in $\mathbb{Z}_p$ so are m_a and m_b. Finally, Alice learns $m = rm_\text{a} - m_\text{b} = r(a_t - b_t)$ for some a_t of Alice's choice. Since r is unknown to Alice this m is uniformly distributed in Alice's view when $a_t \neq b_t$.

Note that if Alice colludes with the server they learn Bob's location. Similarly, if Bob colludes with the server they learn Alice's location.

Performance. The entire protocol requires a small number of additions and one multiplication mod p plus two evaluations of AES. The total message traffic is 12 bytes per edge assuming p is 32 bits. Message encryption adds a few bytes to every message.

Comparing the two protocols. We presented two private equality testing protocols. Which protocol to use depends on the level of trust in the server and the clients' computing power. The first protocol makes no use of a server, but is computationally expensive and requires the two users to communicate synchronously. The second protocol is asynchronous (Bob need not be online when Alice queries the server) which fits better with our application.

In spite of the asynchronicity, due to the need for a counter, each message from Bob can be used in one protocol execution by Alice, which means that we cannot use the optimization where Bob only sends messages when he is moving. In the appendix we present a asynchronous variant of Protocol 1 using an oblivious server that has this additional property. This variant is identical to Protocol 1 in terms of computational efficiency (per invocation). The security properties are similar to protocol 2: a collusion between Alice and the server reveals Bob's location.

All the protocols are asymmetric since only Alice learns the answer. They can be made symmetric (if needed) by executing the protocol twice, once in each direction.

4 Location tags

A *location tag* is a secret associated with a point in space and time. It is a collection of *location features* derived from (mostly electromagnetic) signals present in the physical environment.

Location tagging is a procedure to extract the tag from a point in space-time, together with a comparison or matching function. The matching function can be based on Hamming distance, set distance, etc. The two key properties are:

Reproducibility. If two measurements at the same place and time yield tags τ and τ', then τ and τ' match with high probability. Note that they need not be equal as strings.

Unpredictability. An adversary not at a specific place or time should be unable to produce a tag that matches the tag measured at that location at that time.

To fix intuition, a single location feature can be thought of as having around 10 bits of entropy, whereas the predictability of a location tag is much lower, say 2^{-64}.

Location tags provide a different model for proximity testing. The main advantage is that since the location tags of the two parties need to match, spoofing the location is no longer possible, which stops online brute force attacks.

The main disadvantage is that users no longer have control over the granularity of proximity: the notion of neighborhood is now entirely dependent on the type of location tag considered. For example, with WiFi packets, the neighborhood is defined by the range of the wireless network in question.

Some types of location tags we consider below are more time-varying than others. Note that the location tags studied in the original paper [30] are required to be characteristic a point in space (and time-invariant). The authors need additional modifications to make the tag stable over time. On the other hand, the time-varying nature of the tags is a key benefit for our application.

Now we discuss several possible ways to extract location tags. We have experimentally studied the first 3 categories and confirmed that location tags can be extracted from them, whereas the rest are more speculative.

WiFi: Access point IDs. Wireless access points constantly advertise their presence using a combination of SSID and MAC Address. The former is user-specified while the latter is a 48-bit integer fixed by the manufacturer and unique to each device. Wireless access point IDs are already being used as a geolocation technique by companies such as Skyhook who have created a database of ID-location mappings via "wardriving". The location-tag use-case is somewhat different.

We measured the list of visible MAC addresses from two different (proximate) devices at 3 spots on campus. The differences in hardware led to significant differences in the list of IDs measured. On average, 10.7 IDs were visible per device and the mean number of common IDs was 5.3, i.e., about half. Pruning by signal strength, etc., might help improve this ratio.

WiFi: broadcast packets. The contents of the WiFi traffic rather than just the identities of the nodes offer a much richer potential for extracting location tags. Broadcast packets comprise a variety of different protocols. The source and destination addresses, sequence numbers of packets, and precise timing information all offer varying degrees of randomness.

Protocol	Device 1	Device 2	Common
ARP	1088	1071	832
BROWSER	262	286	255
DHCP	249	237	208
MDNS	600	551	541
NBNS	1134	1190	1117
All	3333	3335	2953

Table 1: Packet counts

We performed an experiment on our university campus to test the rate and quality of location features that can be extracted. To do so, we ignored the traffic that is not restricted to the local network. For example, TCP packets likely originate from external networks, and so an attacker might be able to predict or control the contents of some of those packets, so we ignore them. Of the protocols carrying purely local traffic, we restricted ourselves to the top 5 protocols

by number of packets: 'ARP', 'BROWSER',[5] 'DHCP', 'MDNS', and 'NBNS'. We further limited ourselves to broadcast packets; capturing all packets in "promiscuous mode" assuming that it is supported by the device would yield many more packets.

We logged the packets thus filtered from two different devices over a period of 223 seconds and compared them. We received around 3,300 packets on each, for a rate of around 15.0 packets/sec. Table 1 summarizes the degree of similarity between the two logs.

As we can see, around 90% of the packets are common to both devices. Based on the volume of traffic, we can derive meaningful location tags within 2-3 seconds. Details of how to identify a packet and analysis of the timing correlation between the two devices have been omitted for lack of space.

One problem with using WiFi packets for location tags is that both users need to use the same wireless network. One way to get around this, if the device has multiple interfaces, to execute the protocol multiple times, once for each network. Heuristics such as picking the network with the highest signal strength or the network with the alphabetically lowest MAC address among the networks with signal strength greater than a threshold might be sufficient in practice.

Bluetooth. Like WiFi IDs, Bluetooth IDs are unique to each device. They have the advantage over WiFi IDs of almost always being associated with mobile rather than fixed devices, making Bluetooth-based location tags more time-variable. One concern with Bluetooth is that the range may be too small to be very useful. We performed an experiment in a coffee shop where we divided it into a grid and confirmed that the top 5 IDs by signal strength were visible in most of the locations.

GPS. The Global Positioning System works by timing signals sent by a number of orbital satellites. GPS consists of several signals, including "$P(Y)$" (encrypted precision code) and "M-code" (military). Unlike the civilian code, military codes are designed to be unpredictable without a secret key. Recent work by Lo et al. [21] shows that the composition of $P(Y)$ or M signals from 4 or more satellites forms a secure location tag that is temporaly and spatialy unpredictable by design, as needed for our application. These tags would have a configurable granularity as well.

Unfortunately, since these codes run at a higher frequency than the civilian code (to provide better accuracy), current commercial GPS recievers are too slow to measure them. Lo et al. argue that the next generation Broadcom GPS recievers will have the ability to measure at these higher rates making GPS-based location tags practical.

GSM. In cellular networks, various (GSM) cell towers are in range at any one time. Each tower has space- and time-varying characteristics such as signal strength. We are currently investigating if these can be used as location features.

Audio. Audio might be useful in certain limited circumstances to extract location features. For example, music playing in the background in a coffee shop or a talk in a conference room. Acoustic fingerprinting is a well-known technique to extract features from audio in a way that is robust to transformation of the audio such as compression and adding noise. It is used as a way to identify an audio signal from a database of tracks (e.g., Shazam). Similar feature extraction techniques might enable location fingerprinting.

Atmospheric gases. The cell-phone-as-sensor project has experimented with CO, NOx and temperature sensors on cell phones and hopes this will become standard on smartphones so that it

[5]This refers to Microsoft SMB server and not to web browsing activity.

can be used for real-time pollution mapping and other applications [16, 28]. If these ever become mainstream, they are another potential source of location tags.

4.1 Proximity testing using location tags

One way to formulate the underlying cryptographic problem of private proximity testing using location tags is *private set intersection*. Alice and Bob have sets A and B respectively and wish to privately compute $|A \cap B|$. A and B represent location tags; the players conclude that they are nearby if the size of the intersection exceeds a threshold t.

Private set intersection is a well studied protocol and the best solution we know of was provided by Freedman, Nissim and Pinkas [11]. It makes use of a homomorphic encryption scheme E that anyone can encrypt and Alice can decrypt. At the end of the following protocol, Alice learns $|A \cap B|$ and Bob learns nothing:

- Alice interpolates a polynomial p whose set of zeroes is the set A.
- Alice sends $E(p)$ to Bob, i.e., the sequence of encrypted coefficients
- Bob evaluates $E(p(b))$ for each $b \in B$.
- For each $b \in B$, Bob picks a random r and computes $E(rp(b))$
- Bob sends to Alice a permutation of the encryptions computed in the previous step
- Alice decrypts each value she recieves; she outputs the number of nonzero decryptions as $|A \cap B|$.

Note that there are homomorphic encryption schemes where it is possible to multiply inside encryption by a constant using $O(1)$ modular exponentiations. To see that this protocol works, observe that $rp(b)$ is zero if $b \in A$ and a random nonzero value otherwise.

This protocol has two disadvantages. First, it requires $\theta(|A| \cdot |B|)$ modular exponentiations ($E(p(b))$ can be evaluated using Horner's rule using $O(|A|)$ modular exponentiations, and there are $|B|$ such encryptions to compute). Second, it is only secure against semi-honest players. There is a version that handles malicious players, but it is significantly more complex and even less efficient.

Also, revealing the size of the intersection can be a privacy breach. This is of course a weakness of the problem formulation rather than the protocol. It allows Alice to carry out an *online brute force attack* by incorrectly reporting her input set A. This can be a problem when the individual location features have low entropy.

To avoid these weaknesses, we formulate the problem differently. Specifically, we consider the following relaxation of *private threshold set intersection*: Alice and Bob have sets A and B respectively of n elements each and wish to determine if $|A \cap B| \geq t$. If the condition does not hold, then the parties should learn nothing except the fact that $|A \cap B| < t$. But if it does hold, then no privacy is required: the values of A and B can be leaked. Except for the relaxation of the privacy requirement when $|A \cap B| > t$, the formulation is similar to private threshold set intersection.

The quantities n and t are globally fixed. This is necessary because if these values were allowed to depend on the location, that might itself be a privacy leak. We suggest $n = 20$ and $t = 15$ for a good security level when location features have around 10 bits of entropy.

We are able to construct a very efficient protocol for relaxed private threshold set intersection. This protocol requires making assumptions on the partipants' input distribution, i.e., a model for the randomness of the location tags, which we now describe.

Let X denote the domain of the location tags.

- If Alice and Bob are "nearby," we assume that A and B are sampled as follows: for some $t' \geq t$, $A = C \cup A'$ and $B = C \cup B'$ where $C \in_R X^{t'}, A \in_R X^{n-t'}, B' \in_R X^{n-t'}$. This means that their location tags are random subject to the constraint that at least t of them match.
- Alice and Bob are "apart," A and B are sampled as before for some $t' < 2t - n$.

Note that there is a gap (of $n - t$) between the two conditions above. The protocol makes no guarantees on correctness or security when neither the "nearby" nor the "apart" condition holds. Recall a similar approximation factor in the grid-based protocols. In practice we expect that with good quality location tags, t' will be close to 0 or n.

We now describe a protocol for the asymmetric version of relaxed private threshold set intersection. Here it is Bob who obtains the answer while Alice learns nothing.

Protocol 3.

- Alice encodes her input as a set P of points $\{(p_1, x_1), (p_2, x_2) \ldots (p_n, x_n)\}$ where $p_i \in \mathbb{F}$ and $x_i \in \mathbb{F}$. Similarly Bob encodes his input as a set $Q = \{(q_1, y_1), (q_2, y_2) \ldots (q_n, y_n)\}$.
- Alice constructs a polynomial p of degree $n-1$ defined by the points P. Alice picks a random set of points R on p such that $R \cap P = \{\}$ and $|R| = 2(n - t)$.
- Alice sends R to Bob.
- Bob attempts to find a polynomial p' of degree $n - 1$ that passes through at least $2n - t$ of the points in $Q \cup R$. If he is successful, he outputs 1 otherwise he outputs 0.

Correctness. We state a generalized form of the Berlekamp Massey decoding algorithm from [18]:

Theorem 1 (Berlekamp-Massey decoding) *There is an algorithm BM such that, given k pairs (x_i, y_i) over a field $\mathbb{F}$ and a degree parameter d*

- *if there exists a polynomial p that passes through at least $\frac{k+d}{2}$ of the points, BM outputs p*
- *otherwise BM outputs $\perp$*

The proof of correctness now continues.

Case 1. When Alice and Bob are nearby, there are at least $t + 2(n - t) = 2n - t$ points on the polynomial. Substituting $k = n + 2(n - t) = 3n - 2t$ and $d = n - 1$ in Theorem 1, we find that the condition is satisfied, and therefore Bob succeeds in finding a polynomial.

Case 2. When Alice and Bob are apart, we start with the observation that $|A' \cap B'| < n-t$ w.h.p. (specifically, with probabiliity at least $1 - (\frac{n^2}{N})^{n-t}$). This is a matter of bounding probabilities and we omit the proof. By construction this implies $|A \cap B| < t$. This means that there are fewer than $2n - t$ points on the polynomial p, and by Theorem 1, Bob will fail to find an appropriate polynomial.

This completes the proof of correctness. ■

Security. We show that when Alice and Bob are apart, Alice has information theoretic security. Let us give Bob some auxiliary information: specifically, which of his location features are common with Alice. In spite of this extra information, Bob has $2(n-t)+t' < n$ points from a polynomial of degree $n - 1$, and therefore his view of the protocol is statistically indistinguishable from random.

Using the location itself. As presented, this protocol does not use the location co-ordinates. This might be desirable in practice due to the poor coverage of GPS in indoor environments (where location tags are abundant), or to enable location features on devices such as laptops that do not have GPS capabilities. Alternately, the protocol could be modified to treat the x and y location co-ordinates as two of the location features.

Collusion resistance. This protocol is not secure when multiple instances are run with the two sets of inputs being dependent. Therefore, only location tags that are completely time-varying (i.e., independent in each epoch) will work. Among the types of tags that we have empirically studied, that boils down to WiFi broadcast packets. It also means that the protocol needs to be run synchronously, even though there is only a single message sent.

To achieve collusion resistance, each of Alice's executions of the protocol with her different friends must use different location tags. Given the abundance of broadcast packets in our experiment[6], this is feasible. We can use a hash function to ensure that in each instance a random subset of location features are used. This would work as follows:

The protocol instance between players i and j uses only location features f for which $H(i, j, \eta, f) = 0$ where η is the epoch and H is a hash function modeled as a random oracle with range $\{1, \ldots k\}$, k being (say) 20. This way, if Bob and Carol collude against Alice, Alice is still safe because on average only $\frac{1}{20}$ of the features used in the two protocol instances are common.

However this protocol is not cryptographically composable; therefore if sufficiently many (specifically, $\Omega(k)$) friends collude then security will be lost.

5 Implementation

We have built a prototype implementation on the Android platform. The proximity detector is a "background activity" that alerts the user when one or more friends are nearby. Protocols 1 and 2 have been implemented. In this section we describe some of the implementation details.

On the client we use Bouncy Castle, the standard cryptography toolkit for Android. Unfortunately, Bouncy Castle does not yet support Elliptic Curve Cryptography, so we implemented Protocol 1 over Z_p^* with a length of 1024 bits. We implemented our own server in Python. The server acts as a router in protocol 1; in either protocol, there is no big integer arithmetic on the server.

Grid. We allow the grid size to be user-configurable with pre-defined defaults of $10m$, $100m$ and $1000m$. There is no conflict is the grid sizes of a pair of users don't match. The protocols are asymmetric, and in each instance of the protocol the "receiver" quantizes his location according to the "sender's" grid size. Since the sender gets no information at the end of the protocol, the receiver can do so without worrying about security.

Since the earth is round and the tesellation in discussed Section 2.5 assumes a plane world, we divide the earth into strips (corresponding to 1 latitude each). The curvature of the earth within a single strip is small enough to ignore. This allows us to keep the size of the grid cells approximately the same everywhere, but there is a small error probability at the boundary between two of the strips (because the cells on either side of the strip don't line up).

The functionality defined in Section 2.5 has an undefined behavior when the distance between two users is between δ and $\gamma\delta$. In our implementation we take the straightforward approach of

[6]If there are 300s in an epoch, the number of usable location features is around 4,500.

reporting proximity as long as the users are colocated in at least one of the hexagons. This means that the probability of being detected as nearby drops gradually from 1 to 0 as the separation increases from δ to $\gamma\delta$.

Synchronization. Recall that protocol 1 needs to run every few minutes (epochs) in a synchronized manner. The epoch duration in our implementation is fixed globally at 5 minutes.

All communication is through HTTP. We chose to eschew a "push" infrastructure such as Comet and instead adopted the following solution. For each round, each device first performs computations and sends the results to the server; then it waits for a fixed time interval (30s) from the start of the round for the other devices to finish their send steps. Then the device downloads the data from the server that it is supposed to receive in that round.

5.1 SocialKeys: key exchange over social networks

Conventional PKI using certificate authorities is too heavyweight for the needs of most users. A well known usability evaluation of PGP in 1999 concluded that it is nearly unusable for the majority [40]. While the software has improved since then, the underlying architectural and design issues remain.

SocialKeys embraces the idea that public keys must be associated with the digital identities that people already own and use, i.e., their social network accounts, rather than requiring the creation of new identities for cryptographic purposes. While this is not a new idea, we go one step further by enabling such an association for existing social networks, even though none of them support such a capability. We achieve this not by relying on a third-party service, but rather by repurposing social network features in unintended ways.

By offloading the establishment of trust between users to the social network, SocialKeys obviates the need for a "key manager". As a consequence, it is almost completely transparent to the user.

Currently we have implemented key distribution over Facebook as well as an Android "service" that exposes an API that any SocialKeys-aware applications can use. Support for other popular social networks as well as extensions for Firefox (to enable secure web mail, including Facebook messages) and Pidgin (to enable secure instant messaging) are potential future extensions. We are in discussions with two prominent social networks to investigate the possibility of tighter integration.

Facebook might be particularly suitable for key distribution because the company devotes considerable resources to removing fake or duplicate profiles. In general, social network users gain confidence in their friends' identities by observing their status updates and other activity, through one-to-one communication, and through the transitivity of trust in identities.

However, we want to keep the architecture as general and extensible as possible. Every component of our system is replaceable and it can interoperate with OpenID or another social network.

Architecture of client application

- On the user's first interaction with SocialKeys, the client application generates a public/private key pair. The user is asked for a password; in addition to this password, they are also presented with three random dictionary words to memorize. Each dictionary word has around 16 bits of entropy, and therefore we can get a 80-bit security level starting with a reasonably strong password with around 32 bits of entropy. The password is generated according to PKCS 5 (RFC 2898) [19], with an iteration count of 1000.

  To set up SocialKeys on a new device, the user re-enters the password and dictionary words from their first setup. This ensures that the same key pair is reproduced.

- The public key can be uploaded to the social network in one of two ways: 1. directly, encoded as a URL; 2 by pointing to a key server. Currently only the first method is implemented. An example of such a URL is given below:

  ```
  https://socialkeys.org/pubkey?alg=DH&keylen=1024&p=oakley&g=2&key=LlI+lKCAIE...
  ```

  The latter approach is more complex to implement but has some advantages in that it is more extensible and avoids URL length limits. We decided that it was overkill for our purposes.
- Most social networks allow the user to specify one or more "websites" in her profile. We make use of this feature to store the key.
- The client application lives as an Android background process and exposes an API that allows it to receive the identity of any Facebook user (or any supported identity). On receiving an identity the client downloads the public key of that identity and computes and returns a shared secret key.
- Once an identity has been seen by SocialKeys it is periodically polled and the key is kept up to date. Currently this is the only revocation mechanism. The Facebook API allows retrieving the public keys of all of one's friends with a single query, so it ends up being quite efficient in practice.

5.2 Evaluation

Performance. We calculated the CPU time required to run Protocol 1 and Protocol 2 with 100 (mutual) friends, i.e., the protocol was executed in both directions. Protocol 1 took 46.2 $\pm$ 1.4 seconds and Protocol 2 took 3.0 $\pm$ 0.3 seconds. The device we used was a Motorola Droid, with a ARM Cortex-A8 500MHz processor.

Note that the epoch for Protocol 1 is 5 minutes, so we can easily handle several hundred friends (although the battery life may be a concern; we have not tested this aspect.) Also, we have not implemented any of the optimizations described in Section 3.1 for computing exponentiations. Furthermore, as mentioned earlier Elliptic Curve Cryptography libraries are not yet available on the Android platform. Once we are able to switch to ECC, we expect to be able to see a significant speedup. For Protocol 2, the CPU load is far lower.

Testing other aspects of the system such as usability is planned.

6 Discussion

Proximity testing in the peer-to-peer model. In Section 2, we explained how the client-server model does not allow proximity testing between strangers. Interestingly, the peer-to-peer broadcast model allows proximity testing between strangers, but does not allow limiting proximity testing to friends.

All-pairs proximity testing may be achieved in this model by simply having each node broadcast its identity in the clear to its neighboring nodes. Of course, the node has little or no control over the distance threshold; the set of neighbors is determined by the specifics of the channel used (Bluetooth, WiFi, etc.)

To see why any access control is infeasible, note that if the identity is no longer sent in the clear, then it will need to be separately encrypted for each friend (whether or not the friend is actually nearby). Unlike in the client-server model, each node ends up receiving a broadcast from every adjacent node, making the bandwidth requirement $O(|E|d)$ where E is the number of edges in the social network and d is a degree-bound on the proximity graph.

Thus, a peer-to-peer broadcast might serve as a nice complement to the client-server protocols we have developed – users might be comfortable broadcasting their identity to their neighborhood in some situations but not in others. There are however tricky implementation issues involved in developing robust applications over Bluetooth or WiFi, and we have not attempted to do so.

Leaking the result of proximity testing to the server. In the protocol of Section 3.2, the server remains oblivious to the result of the proximity test. It is tempting to design protocols in which the server does learn the result, because it can be made even more efficient.

However, the outcome of pairwise proximity testing can be much more revealing than is at first apparent, especially when combined with *auxiliary information*. Suppose the server has auxiliary information about the work locations of most of the users. Then it can match the proximity-testing adjacency graph with the adjacency graph of known work locations. Highly robust algorithms for matching/inferring such graphs are known [27, 9].

7 Related Work

Location privacy. A non-technical discussion of the issues around location privacy is provided by Blumberg and Eckersley [5].

Many papers have taken the anonymization approach to location privacy. The work of Gruteser and Grunwald [14] kicked off a long line of papers in this area. Another seminal work is by Beresford and Stajano who introduced "mix zones" [4]. For examples of more recent work, we see [17] and [22].

This approach has several potential limitations including the highly identifying nature of location information [13] and the limited location resolution resulting from the obfuscation or quantization needed for anonymization. At any rate, since the application we have in mind is fundamentally social in nature, pseudonymous identities are not suitable.

A privacy-aware friend locator was studied in [39]. Unfortunately the technique used therein has many flaws including the fact that the server always learns the result of proximity testing.

The work closest to ours is perhaps by Zhong et al [42] who study three protocols for privacy-preserving proximity detection in the two party setting. The main difference is that they require the users to learn the mutual distance, which necessitates computationally expensive cryptography. Their work builds on Atallah and Du who study secure multi-party computational geometry [2].

Location tags were introduced in [31, 30]. There were significant differences from our use of location tags: they studied signals such as Loran which are not available on consumer devices, and they require location tags to be stable with time.

There is a body of work on location-based encryption. Some of it assumes the existence of tamper-proof ("anti-spoof") hardware [36]. Other work such as [34] is more rigorous and involves securely verifying the location of the receiver based on timing or strength of electromagnetic signals.

PET and Private set intersection. Fagin, Naor and Winkler discuss numerous protocols for PET with a focus on protocols that can be executed by humans [10].

Freedman, Nissim and Pinkas described a private set intersection protocol based on homomorphic encryption [11]. For the special case where each party's input is a singleton set, this yields a protocol with $O(1)$ modular exponentiations and communication of a constant number of field elements, assuming Paillier encryption is used. Our protocol 1 is similar, except for a better constant and security against malicious parties.

Many other papers have studied private set intersection: [8, 7, 15]. Juels and Sudan construct "fuzzy vaults" which is a related primitive. Our protocol 3 has similarities to their construction.

Cryptography and social networking. The intersection of cryptography and social networking does not appear to be well-studied in the academic literature. The one major work we are aware of in this area is Persona [3], which is a system that gives users fine-grained control of privacy policies using attribute-based encryption.

FaceTrust is a system for users to prove their identity and attributes in a peer-to-peer manner [37], reminiscent of the Web of Trust paradigm [1]. This kind of system can be complementary to SocialKeys because user trust in their friends' online identities is important in the absence of a certificate infrastructure.

A Facebook application called My Public Key allows users to associate public key with their Facebook account [23]. Of course, the interoperability is limited to Facebook users who also install this application.

There is a draft specification for OpenID key discovery [25]. It appears not to be final and there is no actual support from any vendors yet.

8 Conclusion

Location privacy is an important and growing concern in the real world today. Even problems that appear simple such as privacy-aware proximity testing can prove to be surprisingly tricky. In contrast to previous work, we studied this problem on a firm theoretical footing and presented a variety of cryptographic protocols motivated by and optimized for practical constraints. While we have built a prototype implementation, it remains to be seen if any vendors of location-based services will deploy cryptographic systems in the market.

Several of the techniques we came up with may be of independent interest. Asymmetric private equality testing is a versatile primitive, and in Sections 3.1 and 3.2 we provided new protocols and improvements over previous work. Finally, we plan to promote the adoption of SocialKeys independently of the application to location privacy presented in this paper.

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A An asynchronous variant of prot. 1 using an oblivious server

In Section 3.1 we presented a synchronous private equality test using an additively homomorphic encryption scheme such as ElGamal. The protocol requires both Alice and Bob to be online at the same time. Here we present an asynchronous variant of the protocol using a trusted server. The server learns nothing at the end of the protocol, assuming the server does not collude with Alice.

Let (G, E, D) be a public-key additively homomorphic encryption scheme over a group of size p. We assume Alice has the secret key sk and Bob and the server have the public key pk. Using the notation of Section 3.1 the protocol works as follows:

- Bob's message: at any time Bob sends $E(\mathsf{pk}, b)$ to the server,
- Alice queries the server: At any time Alice sends $E(\mathsf{pk}, a)$ to the server. The server generates a random $r \in \mathbb{Z}_p$ and sends $C \leftarrow E\big(\mathsf{pk},\ r(a-b)\big)$ back to Alice. The server constructs C using the additive property of encryption as in Section 3.1.
- Alice obtains answer: Alice decrypts C using sk and obtains $r(a-b)$ from which she learns if $a = b$, but nothing else.

Security follows as in Protocol 1 as long as Alice does not collude with the server. Interestingly, a single message from Bob to the server can be used for multiple queries by Alice. This was not the case in Protocol 2 where a message from Bob could only be used for a single query by Alice. Consequently, when using this protocol Bob need only send a message to the server when Bob changes location.